

On the nature of the so-called generic instabilities in dissipative relativistic hydrodynamics

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Abstract

It is shown that the so-called generic instabilities that appear in the framework of relativistic linear irreversible thermodynamics, describing the fluctuations of a simple fluid close to equilibrium, arise due to the coupling of heat with hydrodynamic acceleration which appears in Eckart's formalism of relativistic irreversible thermodynamics. Further, we emphasize that such behavior should be interpreted as a contradiction to the postulates of linear irreversible thermodynamics (LIT), namely a violation of Onsager's hypothesis on the regression of fluctuations, and not as fluid instabilities. Such contradictions can be avoided within a relativistic linear framework if a Meixner-like approach to the phenomenological equations is employed.

I. INTRODUCTION

Relativistic irreversible thermodynamics has had a rather peculiar history. The first two proposals on the subject came from C. Eckart [1] in 1940 and from L. Landau and I. Lifshitz in the early fifties [2]. Since both of them have been shown to be particular cases of the so-called 'first order theories' [3], we concentrate here on the former proposal, mainly based on the construction of an energy stress tensor where heat flux is included, thus incorporating heat *in the same status as a mechanical energy*. Moreover it is also built so to satisfy the first law of thermodynamics. It was later shown by W. Israel and coworkers [4, 5] that both theories may lead to transport equations of the parabolic type, therefore violating causality. Also, it has been recently suggested that heat, being a non-mechanical form of energy, should not be included in the stress-energy tensor to be coupled to Einstein's tensor in the context of general relativity [6]. Regarding causality, we will leave additional comments on this interpretation and appropriate status in statistical physics for a future publication. Here we want to concentrate on the role of heat in deriving transport equations.

Indeed, besides these difficulties, during the last two decades of the past century, several papers appeared pointing out another shortcoming of the two theories mentioned earlier namely, that they predict hydrodynamic instabilities in the linear regime [2, 3, 5]. This interpretation is not correct. What they really show is that fluctuations under such conditions do not obey the Onsager's regression assumption. We remind the reader that this hypothesis states that spontaneous fluctuations that occur in a fluid in equilibrium evolve towards the equilibrium state following linear equations for the macroscopic variables [7]. This fact led workers in the field to consider the so-called 'second order theories' [4, 8] as those which correctly solve the two main objections. This approach has now been taken as the basis of a new version

of a modified form for the relativistic hydrodynamic equations suitable to deal with the hot dense matter produced in the Relativistic Heavy Ion Collider, RHIC [9].

In this work we show that the problems mentioned above are caused by the structure of the constitutive equation for the heat flux proposed by Eckart which couples it with the hydrodynamic acceleration. This relation, when introduced as a closure to the system of hydrodynamic equations, yields the so-called generic instabilities. It is important to point out that this coupling, whose thermodynamic meaning has always been questioned, has so far not been supported by experiment [10].

As an alternative, we study the effect of a Fourier-like constitutive equation as can be obtained, for example, from Meixner's formulation of linear irreversible thermodynamics [11, 12]. This theory is based upon the idea that the heat flux is a component of a separate total energy flux [13] and leads to a system of linearized transport equations which correctly describe the evolution of the fluctuations according to LIT. Moreover, the second law of thermodynamics is neatly derived and the stress-energy tensor maintains its canonical form including only state variables.

These results suggest that those attempts seeking to deal with these problems basing their ideas on Extended Irreversible Thermodynamics, EIT [14], may be unnecessary. The results obtained from a Meixner-like approach are free from additional parameters other than the standard transport coefficients. On the other hand, EIT, whose constitutive equations are of the Maxwell-Cattaneo type introduces additional parameters that, on the long run, have to be adjusted. Other difficulties with EIT theories have been extensively discussed in the literature [15].

To pursue our discussion we have structured the paper as follows. In section II the relativistic transport equations are derived from Eckart's proposal for the stress-energy tensor. Section III contains the linearized system and the consequent derivation of the so-called instability of the transverse velocity mode. Final remarks, including the alternative proposal corresponding to a Meixner-like scheme, are in-

cluded in the last section of this work.

II. RELATIVISTIC TRANSPORT EQUATIONS: ECKART'S FORMALISM

The evolution of a relativistic fluid is described by the balance equations considered together with suitable constitutive relations. The continuity equation in relativistic hydrodynamics is a statement of conservation of particles. For a non-reactive fluid and denoting the particle flux as $N^\mu = nu^\mu$, such an equation reads

$$N^\mu_{;\mu} = 0 \quad (1)$$

where n is the particle number density. Here and throughout this work greek indices run from 1 to 4, latin indices from 1 to 3, and a semicolon indicates a covariant derivative. The fluid four-velocity is u^μ , so Eq. (1) can be written as

$$\dot{n} + n\theta = 0 \quad (2)$$

where $\theta = u^\mu_{;\mu}$ and a dot denotes a total time derivative.

The general conservation equation for the stress-energy tensor

$$T^\mu_{\nu;\mu} = 0 \quad (3)$$

encompasses both energy and momentum balances. In Eckart's formalism, $T^{\mu\nu}$ is given by

$$T^\mu_\nu = \frac{n\varepsilon}{c^2}u^\mu u_\nu + ph^\mu_\nu + \pi^\mu_\nu + \frac{1}{c^2}q^\mu u_\nu + \frac{1}{c^2}u^\mu q_\nu \quad (4)$$

where ε is the energy per particle in the comoving frame, p is the local pressure, π^μ_ν is the Navier tensor, q^μ is the heat flux and h^μ_ν is a spatial projector defined, for a $(+++ -)$ signature, as

$$h^\mu_\nu = \delta^\mu_\nu + \frac{u^\mu u_\nu}{c^2} \quad (5)$$

The last two terms in Eq. (4) are identified as the heat flux contributions included in Eckart's formalism as part of the proposed relativistic generalization. Also, these

terms satisfy orthogonality conditions equivalent to those for the viscous dissipation namely,

$$u_\mu \pi_\nu^\mu = u^\nu \pi_\nu^\mu = 0, \quad q_\mu u^\mu = q^\mu u_\mu = 0 \quad (6)$$

It is worthwhile at this point to emphasize that these heat flux terms are not present in the stress tensor in the non-relativistic theory as well as in the relativistic version of Meixner's thermodynamics [13].

Equation (3), after computing the covariant derivative of T_ν^μ given by Eq. (4), using Eq. (2) and the operator $(\dot{}) = u^\alpha()_{;\alpha}$ yields the momentum balance equation

$$\begin{aligned} \left(\frac{n\varepsilon}{c^2} + \frac{p}{c^2} \right) \dot{u}_\nu + \left(\frac{n\dot{\varepsilon}}{c^2} + \frac{p}{c^2} \theta \right) u_\nu + p_{;\mu} h_\nu^\mu + \pi_{\nu;\mu}^\mu \\ + \frac{1}{c^2} \left(q_{;\mu}^\mu u_\nu + q^\mu u_{\nu;\mu} + \theta q_\nu + u^\mu q_{\nu;\mu} \right) = 0 \end{aligned} \quad (7)$$

The evolution of the internal energy is given by the projection of Eq. (3):

$$u^\nu T_{\nu;\mu}^\mu = 0 \quad (8)$$

from which one can obtain the equation

$$n\dot{\varepsilon} + p\theta + u_{;\mu}^\nu \pi_\nu^\mu + q_{;\mu}^\mu + \frac{1}{c^2} \dot{u}^\nu q_\nu = 0 \quad (9)$$

where use has been made of the fact that, from Eq. (6), $u^\nu q_{\nu;\mu} = -u_{;\mu}^\nu q_\nu$ and a similar relation holds for the viscous term. It is convenient to recast this equation in terms of the temperature as state variable instead of ε by means of the local equilibrium assumption. That is, since $\varepsilon = \varepsilon(n, T)$, one can write

$$\dot{\varepsilon} = \left(\frac{\partial \varepsilon}{\partial n} \right)_T \dot{n} + \left(\frac{\partial \varepsilon}{\partial T} \right)_n \dot{T} \quad (10)$$

Using the relations $\left(\frac{\partial \varepsilon}{\partial n} \right)_T = -\frac{T\beta}{n^2 \kappa_T} + \frac{p}{n^2}$ and $\left(\frac{\partial \varepsilon}{\partial T} \right)_n = C_n$ where β is the volume expansion coefficient, κ_T the isothermal compressibility and C_n the specific heat at

constant n , Eq. (9) can be written as

$$nC_n\dot{T} + \left(\frac{T\beta}{\kappa_T}\right)\theta + u^\nu_{;\mu}\pi^\mu_\nu + q^\mu_{;\mu} + \frac{1}{c^2}\dot{u}^\nu q_\nu = 0 \quad (11)$$

Equations (2), (7) and (11), the conservation equations, form an incomplete set. In order to obtain the dynamics of the state variables one has to introduce constitutive relations as closure equations. In order to propose these phenomenological expressions for the fluxes $\pi^{\mu\nu}$ and q^μ , one calculates the entropy production from the entropy balance equation which is obtained by means of the local equilibrium assumption, that is, if the entropy per particle is $s = s(n, \varepsilon)$,

$$\dot{s} = \left(\frac{\partial s}{\partial n}\right)_\varepsilon \dot{n} + \left(\frac{\partial s}{\partial \varepsilon}\right)_n \dot{\varepsilon} \quad (12)$$

Substitution of Eqs. (2) and (9) in Eq. (12) and using the thermostatic relations

$$\left(\frac{\partial s}{\partial n}\right)_\varepsilon = -\frac{p}{n^2T}, \quad \left(\frac{\partial s}{\partial \varepsilon}\right)_n = \frac{1}{T}, \quad (13)$$

yields an entropy density balance equation which can be brought to the following general structure

$$n\dot{s} + J^\nu_{[s];\nu} = \sigma \quad (14)$$

where $J^\nu_{[s]}$ is identified as an entropy density flux and the entropy production is given by the expression

$$\sigma = -\frac{q^\nu}{T} \left(\frac{T_{,\nu}}{T} + \frac{T}{c^2}\dot{u}_\nu\right) - \frac{u^\nu_{;\mu}}{T}\pi^\mu_\nu \quad (15)$$

In order to assure the positiveness of σ , and thus satisfy the second law of thermodynamics, Eckart proposes the following constitutive relations

$$\pi^\mu_\nu = -2\eta h^\mu_\alpha h^\beta_\nu \tau^\alpha_\beta - \zeta \theta \delta^\mu_\nu \quad (16)$$

$$q^\nu = -\kappa h^\nu_\mu \left(T^{,\mu} + \frac{T}{c^2}\dot{u}^\mu\right) \quad (17)$$

where the transport coefficients involved are the bulk viscosity ζ , the shear viscosity η and the thermal conductivity κ . In Eq. (16) τ^μ_ν is the traceless symmetric part of the velocity gradient tensor. Equation (17) deserves a closer look. In it, the first term in parenthesis corresponds to the usual Fourier-type constitutive equation. The second term, which arises from the inclusion of the heat terms in the stress tensor, is not in the canonical form namely, it cannot be considered a thermodynamic force. This objection is equally applicable to the second term of Eq. (15). A complete discussion of this issue can be found in Ref. [6] and will not be repeated here. However this fact is brought to the attention of the reader since, as will be shown in the next section, this term leads to the so-called instability found by Hiscock and Lindblom [3].

III. LINEARIZED EQUATIONS FOR THE FLUCTUATIONS

In this section, we shall study the behavior of the linearized equations which result from Eckart's scheme when spontaneous fluctuations of the state variables occur around equilibrium. These equations arise upon substitution of the constitutive equations given by Eqs. (16-17) into the general conservation equations (2), (7) and (11). Next we linearize by setting $T = T_0 + \delta T$, $u^k = \delta u^k$ ($\delta u^4 = 0$) and $\theta = \delta\theta$, which according to Eq. (2) is equivalent to the condition $n = n_0 + \delta n$. Here the naught subscripts characterize their equilibrium values and δ denotes the corresponding fluctuation. The ensuing process requires a minimum effort. In fact, by simple inspection, one notices that the second bracket and the first three terms in the third bracket in Eq. (7) contain at least quadratic terms in the fluctuations as well as the third and fifth terms in Eq. (11). Therefore, the only terms that introduce a difference with the results obtained in ordinary relativistic hydrodynamics are $u^4 q_{\nu;4}$ in the momentum balance and $q^\mu_{;\mu}$ in the energy balance equations. Both arise from the inclusion of heat in the stress tensor.

The rest of the procedure is straightforward. Using the well known techniques of standard linearized non-relativistic hydrodynamics [16, 17, 18] we first write the linearized equations,

$$\delta\dot{n} + n_0\delta\theta = 0 \quad (18)$$

$$\begin{aligned} & \frac{1}{c^2} (n_0\varepsilon_0 + p_0) \delta\dot{u}_\nu + \frac{1}{\kappa_T} \delta n_{,\nu} + \frac{1}{\beta\kappa_T} \delta T_{,\nu} \\ & - \zeta \delta\theta_{,\nu} - 2\eta \left(\delta\tau_{;\nu}^\mu \right)_{;\mu} - \frac{\kappa}{c^2} \delta\dot{T}_{,\nu} - \frac{\kappa T_0}{c^4} \delta\ddot{u}_\nu = 0 \end{aligned} \quad (19)$$

$$nC_n\delta\dot{T} + \left(\frac{T_0\beta}{\kappa_T} \right) \delta\theta - \kappa \left(\delta T^{,k} + \frac{T_0}{c^2} \delta\dot{u}^k \right)_{;k} = 0 \quad (20)$$

where use has been made of the fact that $p = p(n, T)$.

Equations (18)-(20) are the linearized equations for thermodynamic fluctuations in Eckart's version of relativistic hydrodynamics. The next step is to separate $\delta\theta$ from the transverse velocity. For this purpose we calculate the curl and the divergence of Eq. (19). The second operation yields

$$\begin{aligned} & -\frac{\kappa T_0}{c^4} \delta\ddot{\theta} + \frac{1}{c^2} (n_0\varepsilon_0 + p_0) \delta\dot{\theta} + \frac{1}{\kappa_T} \nabla^2 \delta n + \frac{1}{\beta\kappa_T} \nabla^2 \delta T \\ & - \left(\zeta + \frac{4}{3}\eta \right) \nabla^2 \delta\theta - \frac{\kappa}{c^2} \nabla^2 \delta\dot{T} = 0 \end{aligned} \quad (21)$$

This is a scalar differential equation with unknowns $\delta\theta$ (or δn) and δT . We shall come back to it afterwards. On the other hand, the first operation, recalling that the curl of gradient vanishes, yields

$$\frac{\kappa T_0}{c^4} \delta\ddot{U}_\nu - \frac{1}{c^2} (n_0\varepsilon_0 + p_0) \delta\dot{U}_\nu + 2\eta \nabla^2 \delta U_\nu = 0 \quad (22)$$

where $\delta U_\alpha = \epsilon_{\alpha\nu}^\mu \delta u_{;\mu}^\nu$ are the components of the curl of the velocity field. $\epsilon_{\alpha\nu}^\mu$ is the well known Levi-Civita's tensor. Thus, *this procedure decouples the transverse mode fluctuations from the system of hydrodynamic equations*, yielding an independent

equation. Calculating a Fourier-Laplace transform with transform variables k^ℓ and s respectively, we obtain

$$\delta\hat{\hat{U}}_\nu(k^\ell, s) = \frac{\delta\hat{U}_\nu(k^\ell, 0) \left[\frac{\kappa T_0}{c^4} s - \frac{1}{c^2} (n_0 \varepsilon_0 + p_0) \right] + \frac{\kappa T_0}{c^4} \frac{\partial \delta\hat{U}_\nu(k^\ell, t)}{\partial t} \Big|_{t=0}}{\frac{\kappa T_0}{c^4} s^2 - \frac{1}{c^2} (n_0 \varepsilon_0 + p_0) s - 2\eta k^2} \quad (23)$$

where $\delta\hat{\hat{U}}_\nu$ is the Laplace-Fourier transform of δU_ν . Thus, the exponential growth, or decay, of the transverse component of the velocity fluctuations will depend on the roots of the denominator, namely, the solution of the dispersion relation

$$\frac{\kappa T_0}{c^4} s^2 - \left(\frac{n_0 \varepsilon_0 + p_0}{c^2} \right) s - 2\eta k^2 = 0 \quad (24)$$

Equation (24) has two real roots, namely

$$s = \frac{c^2}{2\kappa T_0} \left[n_0 \varepsilon_0 + p_0 \pm \sqrt{(n_0 \varepsilon_0 + p_0)^2 + 8k^2 \eta \kappa T_0} \right] \quad (25)$$

which is precisely the result the authors of Ref. [3] arrived at in a more laborious way. Notice that, not only the procedure here developed yields the dispersion relation in a more clear, direct way but it also highlights two important points. Firstly, the equation for the transverse mode is completely decoupled from the system. Thus, it is unnecessary to make any assumptions about the direction of the velocity fluctuations as done in that work. The second and most important point we wish to emphasize is the fact that, as a consequence of decoupling of both components of the velocity field, longitudinal and transverse, one can easily identify the source of the exponential growth found in the transverse mode fluctuations. Indeed, for spatially homogeneous perturbations, $k = 0$, Eq. (25) yields a positive root

$$s = \frac{n_0 \varepsilon_0 + p_0}{\kappa T_0} c^2 \quad (26)$$

Equation (26) is a critical result. It clearly indicates that Onsager's assumption of the regression of fluctuations is violated. This is in open contradiction with the tenets of LIT. In this sense, Eckarts's formalism is suspect.

IV. DISCUSSION

The results obtained in the previous section, have motivated the formulation and use of generalized constitutive equations. However, as can be clearly seen by inspection of Eqs. (21-24) its cause is precisely the presence of the acceleration term in the constitutive equation for the heat flux, Eq. (17) which, in turn can be traced down to the phenomenological formalism first introduced by Eckart. The coupling proposed in Eq. (17) is thus responsible for the theory being categorized as unphysical, and thus displaced by others [19].

On the other hand, an alternative proposal for the stress energy tensor, the one consistent with Meixner's theory can be easily shown to yield a system of hydrodynamic equations in which fluctuations behave canonically this instability is absent. Indeed, as discussed in Ref. [13], the stress energy tensor there assumed is

$$T_{\mu}^{\nu} = \rho u^{\nu} u_{\mu} + p h_{\mu}^{\nu} + \pi_{\mu}^{\nu} \quad (27)$$

Equation (27) provides an alternative manner to introduce heat without identifying it as a state variable, consistently with classical thermodynamics. Heat is a form of energy, energy in transit, and must be included in a total energy balance equation. In Meixner's irreversible thermodynamics, heat flux is firstly introduced in a total energy balance given by

$$J_{[E];\nu}^{\nu} = 0 \quad (28)$$

where $J_{[E]}^{\nu}$ is the total energy flux and includes the mechanic and internal energy fluxes as well as the dissipative heat flux

$$J_{[E]}^{\nu} = \rho c^2 u^{\nu} + n \varepsilon u^{\nu} + q^{\nu} . \quad (29)$$

This procedure yields an entropy production in terms of products of thermodynamic

forces and fluxes exclusively,

$$\sigma = -q^\nu \frac{T_{,\nu}}{T^2} - \frac{u^\mu_{;\nu}}{T} \pi^\nu_\mu \quad (30)$$

which is consistent with Clausius' idea of uncompensated heat and motivates a constitutive equation for the heat flux where the acceleration term is absent. Indeed, the constitutive relations in this formalism correspond to laws of proportionality between forces and fluxes which ultimately results in a momentum balance equation where the second derivative of the velocity is absent, and thus the evolution equation for the fluctuation of the velocity variable δU_ν reads

$$\left(\frac{n_0 \varepsilon_0 + p_0}{c^2} \right) \delta \dot{U}_\nu = 2\eta \nabla^2 \delta U_\nu \quad (31)$$

which, performing a similar analysis as the one that lead to Eq. (23) yields the dispersion relation

$$- \left(\frac{n_0 \varepsilon_0 + p_0}{c^2} \right) s - 2\eta k^2 = 0 \quad (32)$$

Equation (32) clearly predicts only an exponential decay in time for δU_ν . This result is compared with the unphysical behavior found in the previous section by analyzing the non-relativistic limit of both results for $k \neq 0$ in Appendix A. As shown in this appendix, the root which generates the exponential growth mentioned above contains the thermal conductivity *even in the non-relativistic limit*. This is completely at odds with classical hydrodynamics and with the fact that the heat terms in Eckart's tensor, Eq. (4), are assumed to be strictly relativistic.

To finish this discussion, we would like to go back to Eqs. (20) and (21), a set of coupled equations for δT and $\delta \theta$ (or δn). To solve this system one has to go to the frequency and wave number representation and transform them into a set of two coupled algebraic equations. With the solutions obtained one can calculate the density-density or temperature-temperature correlation functions. The former one, as well known [16, 17, 18], is related to the so-called dynamic structure factor whose

form, for a simple fluid, is known as the Rayleigh-Brillouin spectrum. It consists of a central, or Rayleigh peak and two symmetrically located peaks known as the Brillouin peaks. Rayleigh's peak has a width in frequency $\Delta\omega \propto D_{th}k^2$ where $D_{th} = \frac{\kappa}{\rho C_v}$ is the thermal diffusivity and $k = 2\pi\lambda^{-1}$ when calculated with Eq. (27). On the other hand, if Eckart's tensor is used, it appears the the Rayleigh-Brillouin expectrum does not exist. This fact would be in open contradiction with experiment. Calculations along this line will be published elsewhere.

Appendix A: NON-RELATIVISTIC LIMIT

In this appendix, the non-relativistic limit of Eqs. (24) and (32) are explored. In such limit, one can identify $n_0\varepsilon_0 \rightarrow \rho_0 c^2$ and further notice that $p_0 \ll \rho_0 c^2$ where ρ_0 is the equilibrium mass density. Introducing these facts in the dispersion relation obtained within Eckart's formalism, Eq. (24), yields

$$\frac{\kappa T_0}{c^4} s^2 - \rho_0 s - 2\eta k^2 = 0 \quad (\text{A1})$$

while Eq. (32) yields the linear equation

$$-\rho_0 s - 2\eta k^2 = 0 \quad (\text{A2})$$

Equation (A1) has two roots which, using a binomial expansion for $8\kappa T_0 \eta k^2 \ll \rho^2 c^4$, are

$$s_1 \simeq -\frac{2\eta k^2}{\rho} \quad (\text{A3})$$

$$s_2 \simeq \frac{\rho_0 c^4}{\kappa T_0} + \frac{2\eta k^2}{\rho_0} \quad (\text{A4})$$

Notice that the first root, s_1 , corresponds to the decaying behavior found within Meixner's approach, Eq. (A2). This behavior, in which the velocity fluctuations decay due to viscous effects is more physical than the one given by the second root

s_2 , which corresponds to the so-called instability referred to in Ref. [3]. This behavior of the velocity fluctuations is due to both viscous and thermal effects, a result which we insist, is untenable in this limit.

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